On the Analysis of α-Ray Photographs.

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(Communicated by Prof. Sir E. Rutherford, F.R.S.—Received October 5, 1922.)

[Plate 2.]

1. *Introduction.*

The beautiful method devised by Mr. C. T. R. Wilson* of making visible the tracks of ionising particles, by the condensation of water drops on the ions left in their trail, has been used in this investigation to study the passage of α-particles through gases. As is well known the tracks of these particles appear in these cloud photographs to be very nearly straight except for occasional sudden bends due evidently to collision with the nuclei of the atoms of gas. These bends occur very much more frequently towards the end of the range where the velocity of the particles is low.

The theory of the occurrence of such close collisions between the α-particles and the nuclei, based upon the assumption of a force varying inversely as the square of their distance apart, has been given by Rutherford† and Darwin‡ and verified by Geiger and Marsden,§ Chadwick|| and others. These investigations confirmed in general the hypothesis but indicated a departure from the law of force when the two particles approach very close. Now it is also certain that at sufficiently great distances from the nucleus, the effective force on the particle must be different from that given by the inverse square law owing to the partial or total screening of the nucleus by the electrons. An investigation of the field at such greater distances can be done most sensitively by using particles of such a low velocity that they will not approach closer to the nucleus than the distance considered. As no particles have been detected with certainty with velocities less than about $0{\cdot}4\ v_0$, where v_0 is the initial velocity of the α-particles from radium c, by either the photographic or scintillation methods, it is necessary to make use of the Wilson expansion system.

The beginning of the study of the tracks of α-particles by this method has

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* Wilson, 'Roy. Soc. Proc.,' vol. 87, p. 277 (1912).

† Rutherford, 'Phil. Mag.,' p. 669 (1911) and p. 538 (1919).

‡ Darwin, 'Phil. Mag.,' p. 500 (1914).

§ Geiger and Marsden, 'Phil. Mag.,' vol. 25, p. 604 (1913).

|| Chadwick, 'Phil. Mag.,' p. 923 (1921).

been recently described by Shimizu.* The present work, which has been carried out under the direction of Sir Ernest Rutherford, is a continuation of this and has already led to some interesting results. A statistical study of the frequency of the sudden bends in the tracks in both air and argon has shown that the velocity of the particles very close to the end of the track is as low as $v_0/20$, while the observed *distribution* of bends has been found to be consistent with the hypothesis that the inverse square law of force still holds between the nuclei and the particles when their distance apart is of the order of 10^9 cm. These results do not support the view that collisions of anomalous type and frequency occur towards the end of the tracks as has recently been suggested by Shimizu.†

In addition the study of the rare occasions on which an α-particle sets an atomic nucleus in rapid motion, giving rise to a branched track, has given interesting information about the dynamics of atomic collisions. The collisions considered appear to be perfectly elastic, no loss of energy or momentum being discovered. However this part of the investigation will be discussed in a later paper and our attention at present directed only to the statistical investigation of the deflections of the α-particles themselves.

2. *General Method.*

The procedure outlined by Shimizu has been followed closely in this work, and consisted in taking a large number of photographs of α-ray tracks, using a modified Wilson apparatus and a special camera, which by means of a system of mirrors gives on one film two images of the object viewed from directions mutually at right angles. The way in which the actual form of the track is calculated from measurements of the two images will be described. The result is a knowledge of the frequency of occurrence of bends of different angle at different distances from the end of the track, which is then compared with that to be expected theoretically.

Before considering this theoretical aspect, a short description will be given of the apparatus and its method of use.

3. *The Expansion Chamber.*

The expansion chamber used is one built for Mr. Shimizu, but with the reciprocating mechanism replaced by a simple spring action. This shares with Mr. Wilson's original design the advantage of obtaining very sudden expansions which can be repeated as frequently as required. The arrangement is shown in fig. 1. The speed of the expansion is regulated by the

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* Shimizu, 'Roy. Soc. Proc.,' vol. 99, p. 425 (1921).

† Shimizu, 'Roy. Soc. Proc.,' vol. 99, p. 432 (1921).

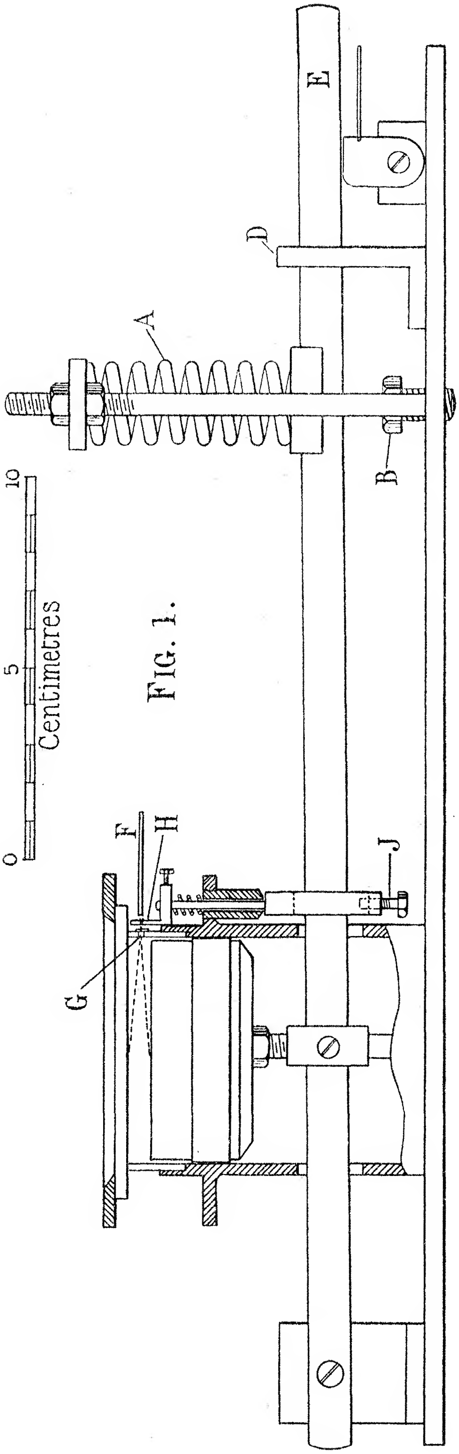


FIG. 1.

tension on the spring A and the amount of the expansion by the adjustable screw B. A paper scale on the bracket D, serves to measure the range of the bar E, and so the expansion ratio. The use of the expansion chamber and the coating of the internal glass surfaces with gelatine has been sufficiently described by Wilson and Shimizu. The source consists of a thin copper wire F, with a polonium deposit on its end. This is held in a carrier (not shown) opposite a small hole G in the glass cylinder wall, which is covered by a thin mica window of about 1 cm. stopping power. Between the source and the window is a mechanically operated shutter, H, worked by the main bar. An adjustable screw, J, allows the shutter to be so arranged that the stream of α-particles can only enter the chamber at the required moment, that is at or very near the bottom of the stroke. This keeps the chamber clear of all but sharp tracks. The stream of α-particles is restricted by means of suitable diagrams to a narrow cone, the axis of which is a diameter of the chamber. In order to facilitate the geometrical reduction of the photographs, the solid angle of this cone is kept as small as possible. The dotted lines in the figure show roughly the extent and position of the beam of rays.

The chamber is illuminated by two arc lamps placed symmetrically on either side, with suitable condensing lenses, heat-absorbing cells, and screens to limit the beams. By this means the two images in the camera are made equally bright.

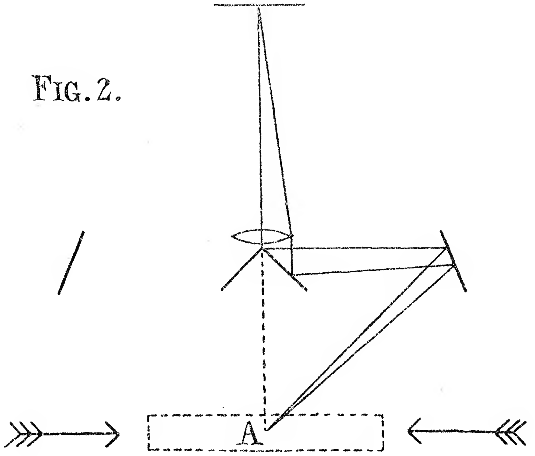
4. *The Camera.*

Shimizu (*loc. cit.*) described the optical arrangement of the camera, showing how the two images viewed from mutually perpendicular directions are projected through a single lens on to the film. His diagram is reproduced in fig. 2, but with the position of the arc lamps added so as to show the symmetrical arrangement. The camera is so placed over the expansion chamber that the line of intersection of the two focal planes is coincident with the axis of the cone of α-rays. That is, this axis would appear on fig. 2 as a line through A at right angles to the plane of the paper. The only modification of Shimizu's arrangement is the method of working the shutter. This is done by means of a Bowden wire, actuated by the main bar of the chamber, and adjusted so that the photograph is taken just as the expansion is completed. This has the advantage that the camera can be moved about as required and yet the shutter worked with the correct timing.

5. *Experimental Method.*

The apparatus is set up as has been described, and then the three adjustments made which are necessary to get good tracks. These are the expansion ratio, the speed of the expansion and the timing of the shutter which uncovers the source. This last should occur just before the bottom of the stroke. The speed of the expansion can be varied within fairly wide limits, but the expansion ratio must be very carefully adjusted. It was shown by Wilson* that if v_1 is the initial and v_2 the final volume, then an expansion for which v_2/v_1 has the value 1·25 is necessary to produce the first rain-like drops in air, while a value 1·38 marks the beginning of the general fog. The value 1·33 was given as that which gives the best tracks. These results appear to apply to all diatomic gases expanded so rapidly that the conditions can be considered as adiabatic.

FIG. 2.



Now Wilson concluded that the type of cloud obtained depended only on the supersaturation at the end of the expansion, and therefore on the value of the ratio of the temperatures θ_1 and θ_2. Now this ratio for a gas, the ratio of whose specific heats is γ, is given for an adiabatic expansion by the relation

$$\frac{\theta_1}{\theta_2}=\left(\frac{v_2}{v_1}\right)^{\gamma-1}.$$

We can calculate, using this relation, the expansion necessary to obtain a given type of cloud for any value of γ, using the values of v_2/v_1 for air given above. In this way we find that, for monatomic gases, expansion ratios of 1·14 and 1·21 are necessary to give the first drops and the fog respectively.

Now, as there is no convenient way of measuring directly the expansion in use, we use these values given by Wilson for air, to calibrate the scale on the bracket D, which measures the relative travel of the bar. In this way the ratio v_2/v_1 in any experiment is determined with sufficient accuracy. It should be noticed that the marked difference between the requisite values of

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* Wilson, *loc. cit.*

v_2/v_1 for monatomic and diatomic gases, affords a means of estimating the proportions of a mixture. This was seen very prettily in one experiment with argon, when a leak in the expansion chamber, letting in air, caused a gradual change in the expansion necessary to give good tracks.

When good tracks have been obtained the camera is placed in position and as many expansions made and photographs taken as desired. The photographs are then all examined, and whenever a track is seen to make a sudden bend, the two images on the photograph are measured up by means of a low-power microscope fitted with an eye-piece scale and cross-wire. A diagrammatic picture of the photograph of such a track is shown in fig. 3. The two images are shown slightly separated as they are actually in practice.

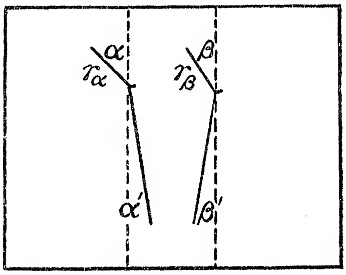

FIG. 3.

Since the information we desire from such a photograph is the angle ϕ of the bend and its distance r from the end of the track, the measurements we make are of the four angles marked α, β, α', β', and the two lengths r_α and r_β. The dotted reference lines from which the angles are measured are at right angles to the edge of the film, and are parallel to the image of the line of intersection of the two focal planes, which latter is the axis of the cone within which the stream of α-particles is projected.

It is now necessary to digress somewhat in order to show how from these six measured quantities we can obtain the actual deflection ϕ, and the distance r from the end of the track at which it occurred.

6. *Geometrical Theory.*

The two images are projections of the track upon two planes at right angles. If we call the line of intersection of these two planes Oz, and a line in each plane at right angles to Oz, Ox, and Oy respectively, we obtain a set of rectangular axes, from the origin of which we can draw two lines OP and OQ parallel to the actual directions of the initial and final parts of the track. Then the angles between OZ and the projections of OP and OQ on the Zx and Zy planes respectively, are the four angles α', β', α, β which are measured on the photographs, as can be seen from fig. 4.

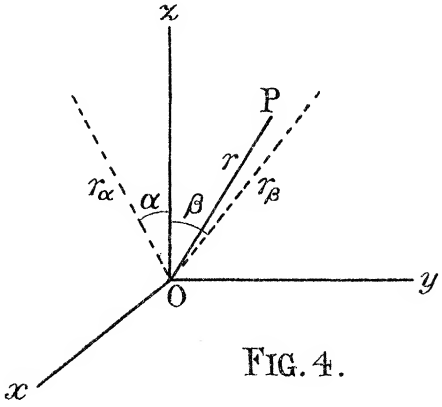

FIG. 4.

If the direction cosines of OP and OQ are l', m', n', and l, m, n, and if the angle POQ is denoted by ϕ, we find, using the relation

$$\cos\phi = ll' + mm' + nn',$$

and the four relations

$$l/m = \tan\alpha, \ \dots \text{ etc.},$$

that

$$\cos^2\phi = \frac{(\tan\alpha\tan\alpha' + \tan\beta\tan\beta' + 1)^2}{(\tan^2\alpha + \tan^2\beta + 1)(\tan^2\alpha' + \tan^2\beta' + 1)}, \qquad (1)$$

which is the expression we require to calculate ϕ.

To calculate the length r of the line OQ, of which the projections on the planes Zx and Zy are r_α and r_β, we have

$$r^2 = r_\alpha^2 + r_\beta^2 \sin^2\beta = r_\beta^2 + r_\alpha^2 \sin^2\alpha, \qquad (2)$$

which give

$$r = k_\alpha r_\alpha = k_\beta r_\beta, \qquad (3)$$

where

$$k_\alpha = \frac{(1 - \sin^2\alpha\sin^2\beta)^{1/2}}{\cos\beta}; \quad k_\beta = \frac{(1 - \sin^2\alpha\sin^2\beta)^{1/2}}{\cos\alpha}. \qquad (4)$$

The equations (3) and (4) are very convenient, since the values of k_α (or k_β) can be read directly from a set of curves of constant k_α drawn for all values of α and β. And this is the method always used, except for the cases when

$$\alpha = \beta = \pi/2,$$

when equation (4) becomes indeterminate and resort has to be made to equation (2).

Returning to the calculation of ϕ, it will be seen that equation (1) is of an inconvenient form, especially when a large number of tracks have to be examined, and although it is used in this investigation when special accuracy is desired, two other methods have been devised to simplify the calculations.

The first of these is based on the fact that, owing to the experimental arrangements, the angles α' and β' are small, except on the comparatively rare occasions when the bend being considered is subsequent to another bend of large angle.

Now for

$$\alpha' = \beta' = 0$$

(1) reduces to

$$\tan^2\phi = \tan^2\alpha + \tan^2\beta, \tag{5}$$

while, when all four angles are *small*, ϕ is given approximately by the relation

$$\phi^2 = (\alpha - \alpha')^2 + (\beta - \beta')^2,$$

which to the same order of accuracy can be written

$$\tan^2\phi = \tan^2(\alpha - \alpha') + \tan^2(\beta - \beta'), \tag{6}$$

which is of the same form as (5), with $(\alpha - \alpha')$ and $(\beta - \beta')$ written for α and β.

It is possible to investigate analytically the errors involved in using (6) instead of (1), but it is simpler to compare the two empirically in a number of cases. This has been done, and it has been found that this error is small when α' and β' are small, provided that α and β are not larger than about 45°.

Equation (6) is convenient, since ϕ is given as a function of the two variables $(\alpha - \alpha')$ and $(\beta - \beta')$, and can therefore be obtained graphically. For this reason this method has been employed in all cases when the conditions given above are fulfilled.

Now there is yet another way in which ϕ can be obtained, which is of special importance when α and β are equal to 90°; for in this case equation (1) becomes indeterminate, and the values of r_α and r_β have to be utilised to calculate ϕ. A sphere is ruled, with two intersecting sets of meridians at regular intervals, of perhaps 10°, from poles which are 90° apart. A photograph of such a sphere is shown on Plate 2, and the diagrammatic fig. 5 shows at once the method of use. In this figure the two polar diameters are called the axes of x and y, and the diameter at right angles to these the axis of z. Then any direction OP can be specified by a point P on the surface of the sphere, and this point P lies at the point of intersection of the meridians marked α' and β'. Similarly, any other direction OQ is specified by the two meridians α and β. And the angle POQ is given by the length of the arc PQ, which can be directly measured.

This method is very simple to use, and has the considerable advantage that it reveals at once the order of accuracy of the angle ϕ when obtained from any given set of measurements. For this will greatly depend upon the particular values of the four angles measured. An approach to the condition when ϕ is indeterminate will be revealed by the decrease of the angle between the two meridians which fix the point. For the less this angle, the greater clearly the indefiniteness in the position of the point.

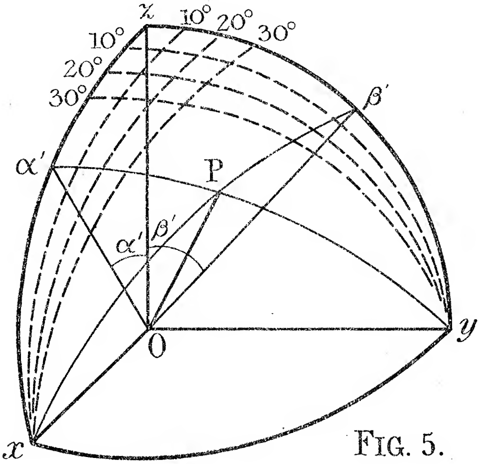

FIG. 5.

In this case of complete or approaching indeterminateness of ϕ, as calculated from the four angles alone, the observed value of r_α/r_β can be used in addition to fix the point Q. In fact, the sphere can be graduated in addition with curves of constant r_α/r_β, since this quantity is a function of α and β only. If this is done,* *all six* observed quantities can be utilised in the determination of ϕ, and suitable *weight* can be given to each of the variables.

The sphere was actually used in the *statistical* work on all occasions when the conditions for the use of equation (6) were unfulfilled.

For the suggestion of thus using the surface of a sphere to simplify the geometrical calculations, I am indebted to Mr. D. R. Hartree, of St. John's College.

7. *The Method of Using the Results of the Experiments.*

We obtain, as a result of these measurements and the subsequent calculations, a knowledge of all the noticeable bends made by a given number of α-particles. Enlarged reproductions of some of the photographs obtained are shown on Plate 2. Numbers 2 to 5 are of tracks in argon, and numbers 6 to 11 in air. The identification of the two images of a given track is assisted by noting that the z co-ordinate of any point, that is the distance from the bottom of the photograph, is the same in both images.

In particular, the end of both images of any track will be found on the same horizontal line. A good example of this is seen in number 6, in which five tracks appear, all ending at different distances (marked by arrows) from the bottom of the photographs. In the description of the photographs, bends will be described by writing the appropriate values of ϕ and r in brackets, ϕ being measured in degrees and r in divisions. Thus (20°, 42) means a bend of 20° at a distance of 42 divisions from the end of the track.

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* The sphere shown in the plate is not provided with these graduations.

The simplest way of representing these results is to mark a point for each bend on a diagram on which ϕ and r are measured along two axes at right angles. In fig. 6 is shown a reproduction of part of such a diagram representing the results of the experiments with argon.

Certain cases will arise when two or more bends are made by a single α-particle, and in this case, strictly speaking, the deflection of the particle at any but the last of these bends should be plotted against the range that the particle would have travelled if the subsequent collision had not been made. But it will be seen from the theoretical discussion that it is legitimate to neglect such a complication. But it must be remembered that not the whole of such a diagram is physically significant, since bends of small angle, though relatively frequent, are not noticed. Thus the density of dots falls off rapidly for small value of ϕ, and as small angle bends are most easily noticed and measured at some distance from the end of the tracks, the limit ϕ_m, above which the observed distribution can be said to have any physical significance, will be smaller for the higher values of r.

Towards the end of the track, where bends occur fairly frequently, an appearance of general curvature is sometimes found. In most cases such an apparent curvature can be resolved into a succession of sharp bends, but even when this is not the case, it is still usually possible to decide whether or no a track has made a bend greater than some given angle.

It is now necessary to derive expressions for the distribution to be expected theoretically.

8. *The Theory of the Collisions.*

Rutherford and Darwin have given the theory of the frequency of collisions between α-particles and the nuclei of atoms, assuming the inverse square law and the conservation of kinetic energy and momentum during a collision. In order to adapt this theory to the present case, it is also necessary to assume that an α-particle of given velocity will travel a definite distance before losing its ionising power. This means that for all α-particles the remaining range r and the velocity v are connected by the same relation,

$$v = g(r).$$

Concerning this last assumption, it has been shown by both Darwin* and

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* Darwin, 'Phil. Mag.,' p. 901 (1912).

Flamm* that the loss of energy of an α-particle to the nuclei of the atoms through which it is passing is on the average small compared to the loss to the electrons. The probability of variations in the loss of energy to the electrons has been calculated by Bohr† and Flamm, and shown to be small. Thus, on the average, the α-particles should behave sufficiently similarly to justify the assumption.

Though occasionally large angle bends do occur, in which a large proportion of the energy is lost to a nucleus, these cases do not appreciably affect the average behaviour of the particles. This assumption, however, is made less sure by the results of some recent work on straggling, which will, however, be discussed later.

Let M, E, v and m, e, u be the mass, charge and final velocity of the α-particle and nucleus respectively. Then if V be the initial velocity of the α-particle and ϕ and θ the deflection of the α-particle and the angle between the direction or the final motion of the nucleus and the initial direction of the α-particle respectively, then we have

$$\begin{aligned} \mathrm{MV} &= \mathrm{M}v\cos\phi + mu\cos\theta, \\ 0 &= \mathrm{M}v\sin\phi - mu\sin\theta, \\ \mathrm{MV}^2 &= \mathrm{M}v^2 + mu^2. \end{aligned}$$

From these we derive, following the analysis of Darwin, but writing η for M/m,

$$v/\mathrm{V} = f(\phi), \tag{7}$$

and

$$\tan^2\theta = \frac{m^2}{(\mathrm{M}+m)^2}\,\mathrm{F}(\phi), \tag{8}$$

where

$$f(\phi) = \frac{\eta\cos\phi \pm (1-\eta^2\sin^2\phi)^{1/2}}{1+\eta},$$

and

$$\mathrm{F}(\phi) = \operatorname{cosec}^2\phi\,[\cos\phi \pm (1-\eta^2\sin^2\phi)^{1/2}]^2.$$

If p be the distance from the initial position of the nucleus to the line of approach of the α-particle, Darwin showed that the calculation of the orbit gives

$$p = \tan\theta\,\frac{e\mathrm{E}}{\mathrm{V}^2}\left(\frac{1}{\mathrm{M}} + \frac{1}{m}\right). \tag{9}$$

If there are N atoms of gas in unit volume, the probability, n, that a single particle will make a collision for which ϕ is less than some given value, when travelling a distance dr, is given by

$$n = \pi p^2 \mathrm{N}\,dr. \tag{10}$$

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* Flamm, 'Wien. Ber.,' IIa, vol. 123, p. 1393 (1914).

† Bohr, 'Phil. Mag.,' vol. 30, p. 581 (1915).

In such a collision the particle is deflected through an angle greater than ϕ. From (9) and (10) we get

$$n = \pi N dr \frac{e^2E^2}{V^4}\left(\frac{1}{M}+\frac{1}{m}\right)^2 \tan^2\theta,$$

which using (8) leads to

$$n = \frac{K}{V^4} F(\phi) dr, \tag{11}$$

where

$$K = \pi N e^2 E^2 / M^2.$$

At this point Darwin's analysis becomes inapplicable to the present case, which requires the following procedure.

Differentiating (11) we get

$$dn = \frac{K}{V^4} F'(\phi)\, d\phi\, dr, \tag{12}$$

which gives the probability that an α-particle of velocity V makes a collision for which ϕ lies between ϕ and $\phi + d\phi$ when travelling a distance dr. In any collision of this type the particle will have its velocity reduced in the ratio v/V, but since the total number of collisions in which this ratio is appreciably less than unity is very small, the average velocity of the α-particles over the distance dr can be considered as unaltered.

Thus from (7) and (12) we obtain

$$dn = \frac{K}{v^4} F'(\phi)\,[f(\phi)]^4 d\phi\, dr. \tag{13}$$

This gives the probability of a collision in which the *final* velocity is v, and so after which the particle has a fixed remaining range r, given by

$$v = g(r).$$

Thus (13) becomes

$$dn = K\, F'(\phi)\,[f(\phi)/g(r)]^4 d\phi\, dr,$$

and the average number $\bar{P}$ of α-particles which are deflected through angles lying between ϕ_1 and ϕ_2 with remaining ranges lying between r_1 and r_2 will be given by

$$\bar{P} = QK \int_{\phi_1}^{\phi_2} \int_{r_1}^{r_2} F'(\phi)\,[f(\phi)/g(r)]^4 d\phi\, dr, \tag{14}$$

where Q is the total number of particles considered.

It will be seen that $\bar{P}$ is also the average number of dots which may be expected to be found in a rectangular area or cell of the distribution diagram limited by values ϕ_1, ϕ_2, r_1, r_2. From (14), supposing the right-hand side to be completely known, we can find $\bar{P}$ for all the n cells into which it is convenient to divide the distribution diagram. The actual numbers found in these cells can be counted. A consideration of the relation to be expected between the average distribution given by the theory and the actual observed distribution, which will of course be liable to chance variations, will occupy the next section of this paper.

9. *The Chance Variations in the Frequency of Bends of Given Type.*

It should be noted that the physical meaning of the average number $\overline{P}_r$ in the rth cell is that *average* number which would be found in that cell, in a very large number of separate experiments, all with the same total numbers of tracks, and all conducted under such circumstances that the theoretical laws of collision are obeyed.

Since we are supposed to be dealing with a large number of *dots,* we can consider the total number, S, to be the same in each experiment, in spite of the fact that the number is actually liable to chance variations.

Then all sets of numbers $P_1, P_2, P_3, \ldots P_n$, consistent with the equation

$$\Sigma\, P_r = S,$$

give possible, but not equally probable, distributions.

I am indebted to Mr. C. G. Darwin for having shown that the mean square departure of the observed numbers P_r for the rth cell from the average number $\overline{P}_r$ is, when all possible distributions are considered, given by

$$\overline{(P_r-\overline{P}_r)^2} = \overline{P}_r\left(1-\frac{\overline{P}_r}{S}\right).$$

If now we divide by $\overline{P}_r$, and sum over all the n cells, we find that

$$\Sigma\, \frac{\overline{(P_r-\overline{P}_r)^2}}{\overline{P}_r} = \Sigma\left(1-\frac{\overline{P}_r}{S}\right) = n-1,$$

since

$$\Sigma\, \overline{P}_r = S.$$

If then we write for every distribution,

$$F = \Sigma\, \frac{(P_r-\overline{P}_r)^2}{\overline{P}_r},$$

the *average* value of F for all possible distributions is given by

$$\overline{F} = n-1.$$

If, then, for any given distribution F has a value not far from $(n-1)$, we conclude that the distribution in question is an average one. But if F has a value greater than, say, $3n/2$, then the observed distribution is improbable and differs markedly from the average. But for physical purposes we can interpret a value of F about equal to $(n-1)$ as indicating that the *theoretical* *distribution utilised is the right one,* while a markedly larger value indicates that the *theoretical distribution is wrong.* The value of F thus appears as a criterion for the correctness of the theoretical average distribution.

However, it is not possible to calculate F for the observed distribution, since the right-hand side of (14) contains the unknown function $g(r)$. But the integration with respect to ϕ can readily be carried out, giving

$$\overline{\mathrm{P}} = \mathrm{QKS}(\phi_1, \phi_2)\int_{r_1}^{r_2}[g(r)]^{-4}\,dr, \tag{15}$$

where

$$\mathrm{S}(\phi_1, \phi_2) = \int_{\phi_1}^{\phi_2}\mathrm{F}'(\phi)\,[f(\phi)]^4\,d\phi.$$

We can now compare the observed and expected distributions by confining our attention to an area of the distribution diagram bounded by two values, r_1 and r_2. This area will cover a range of ϕ from 180° to ϕ_m, which latter will mark the end of the significant part of the diagram.

Equation (15) gives the relative numbers to be expected in each of the cells into which this area can be divided. Their absolute values can be found approximately by putting the expected number, $\overline{\mathrm{P}}$, for the *whole area* equal to the observed number, P. If P is large this can be done without any large fractional error. Then the expected numbers for each cell can be calculated from (15), and the value of F for the distribution obtained. If this is not much greater than $(n-1)$, we conclude that the theoretical distribution is right. This being so, it is legitimate to proceed and calculate the velocity of the α-particle.

10. *The Calculation of the Velocity.*

Equation (15) gives for the area under consideration

$$\int_{r_1}^{r_2} v^{-4}\,dr = \frac{\mathrm{P}}{\mathrm{QKS}(\phi_m, \pi)},$$

where, as before, P has been written for $\overline{\mathrm{P}}$.

If y be written for v^{-4}, and if $\bar{y}$ is its mean value over the range r_1 to r_2, then

$$\bar{y}(r_2-r_1) = \frac{\mathrm{P}}{\mathrm{QKS}(\phi_m, \pi)}, \tag{16}$$

and it is clear that there will be some value, r_0, between r_1 and r_2 for which y has its mean value. This value will be given by the solution of the equation

$$\int_{r_1}^{r_2}[g(r)]^{-4}\,dr = [g(r_0)]^{-4}[r_2-r_1]. \tag{17}$$

The procedure, then, for obtaining the function $g(r)$ from the distribution diagram is to evaluate $\bar{y}$ for each separate area, using (16), and by plotting the values so obtained against the values of $\frac{1}{2}(r_1+r_2)$ to obtain an approximate expression for $g(r)$. This is then used in (17) to give approximate values for r_0, the use of which instead of $\frac{1}{2}(r_1+r_2)$ gives a second approximation to $g(r)$. This process can be repeated as is found necessary.

It is interesting to note that in the special case of $g(r)$ being linear, that is of the form $(A+Br)$, the solution of (17) is

$$r_0'^4 = \frac{3r_1'^2 r_2'^2}{r_1'/r_2'+r_2'/r_1'+1}, \tag{18}$$

where $r' = r+A/B$, etc.

This simple result, which is very useful as a first step in the approximation, shows at once that r_0 always lies on the low velocity side of the mid-point of the area. For from (18)

$$r_0' < (r_1' r_2')^{1/2} < \tfrac{1}{2}(r_1+r_2).$$

That this should be so is otherwise obvious.

In the actual calculation of the velocity for each area, it is convenient to measure r_1 and r_2 in divisions of the scale of the microscope, used in the measurements of the photographs. If there are q such divisions to 1 cm., and if p and M denote the expansion ratio and magnification of the camera respectively, while T and ρ are the initial temperature and pressure of the gas, it can be seen that the velocity is given by

$$v = K_1 K_2 w,$$

where

$$K_1^4 = \pi N_0 e^2 E^2/M^2,$$

$$K_2^4 = \rho T_0/\rho_0 T p q M,$$

and

$$w^4 = QS(\phi_m, \pi)(r_2-r_1)/P.$$

And it will be seen that while K_1 depends on well-established atomic constants only, K_2 involves all the directly measured quantities, and w is given by the results of the statistical investigations.

The values of $S(\phi_1, \phi_2)$, which are required for the experiments with air and argon now to be described, are given below. For the former, the value of η is $1/3{\cdot}6$ and the integration was done graphically. For argon, η is equal to $1/10$, and the integration was carried out term by term, after expansion of the integrand in terms of ascending powers of η. It can easily be seen that for η equal to zero $(S\phi_1, \phi_2)$ reduces to $(\cot^2\phi_1/2-\cot^2\phi_2/2)$, which is the expression given by Rutherford. The values tabulated are for $S(\phi, \pi)$, from which the values of $S(\phi_1, \phi_2)$ can be obtained by subtraction. The values of $\cot^2\phi/2$ are given for comparison.

Table I.

	ϕ.	5°.	10°.	15°.	20°.	25°.	30°.	35°.	40°.	60°.
$\eta = 1/3{\cdot}6$	$S(\phi, \pi)$	524	121·5	47·4	27·0	16·4	10·4	6·8	4·4	1·1
$\eta = 1/10$	$S(\phi, \pi)$	524	127·1	54·8	29·7	18·1	12·0	8·5	6·1	2·1
$\eta = 0$	$\cot^2\phi/2$	524	130·7	57·8	32·2	20·3	13·9	10·1	7·6	3·0

11. *The Experiments with Argon.*

The first actual experiments were carried out with a mixture of 75 per cent. argon and 15 per cent. nitrogen, since the high atomic number of argon, compared with air, results in many more bends being made by the α-particles. The distribution of bends made by the 281 tracks which were considered is shown on fig. 6. This diagram was divided for analysis into four areas, the

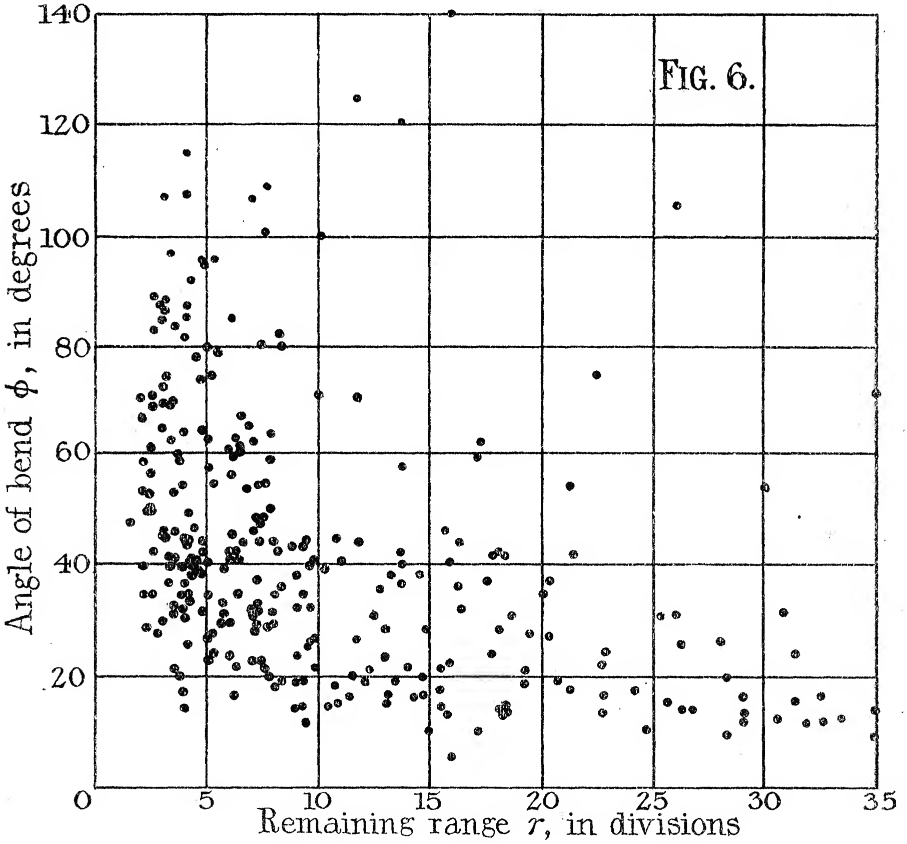
Fig. 6.

limits of which were 3 to 7, 7 to 15, 15 to 30, and 30 to 100 divisions.* Within each area the number of dots P between given limits ϕ_1 and ϕ_2 were counted, and are shown in Table II. Below each number P is the expected number $\bar{P}$, obtained as described above, by putting $\bar{P}$ for the whole area from π to ϕ_m equal to P for this area, ϕ_m being the limit of ϕ beyond which the distribution is clearly not in agreement with the theory. The values of P below this limit are enclosed in brackets.

Table II.

<table>
<tr><th>ϕ_1, ϕ_2.</th><th>20, 30.</th><th>30, 40.</th><th>40, 50.</th><th>50, 60.</th><th>60, 70.</th><th>70, 80.</th><th>80, 90.</th><th>90, 180.</th></tr>
<tr><td>$r_1 = 3$ P ....</td><td>(11)</td><td>(20)</td><td>(22)</td><td>(8)</td><td>13</td><td>7</td><td>6</td><td>8</td></tr>
<tr><td>$r_2 = 7$ $\bar{P}$ ....</td><td>18·5</td><td>62·4</td><td>27·7</td><td>14·2</td><td>12·3</td><td>7·5</td><td>4·7</td><td>9·9</td></tr>
<tr><td>$r_1 = 7$ P ....</td><td>(21)</td><td>(17)</td><td>16</td><td>5</td><td colspan="3">7</td><td>5</td></tr>
<tr><td>$r_2 = 15$ $\bar{P}$ ....</td><td>71</td><td>23·6</td><td>10·4</td><td>5·4</td><td colspan="3">6</td><td>2·4</td></tr>
<tr><td>$r_1 = 15$ P ....</td><td>(12)</td><td>8</td><td>7</td><td>2</td><td colspan="4">5</td></tr>
<tr><td>$r_2 = 30$ $\bar{P}$ ....</td><td>32·8</td><td>10·9</td><td>4·8</td><td>2·5</td><td colspan="4">3·8</td></tr>
</table>

ϕ_1, ϕ_2.	10, 20.	20, 30.	30, 180.
$r_1 = 30$ P	20	3	3
$r_2 = 100$ $\bar{P}$	19·9	3·6	2·4

It will be seen that there is a rough agreement between the two sets of numbers. Further, since the value of F for the 16 cells to which the numbers relate is found to be 11·3, we can conclude that the observed distribution is consistent with that expected theoretically.

An objection may be made as to the arbitrary nature of the choice of ϕ_m. The justification for the procedure lies in the fact that the observed numbers in all cases agree with those expected for the higher values of ϕ, while they are invariably less for small values. This reduction in apparent frequency may be real, and due possibly to electronic screening, but many more experiments would be needed to establish it with certainty. The measurements on the actual photographs were carried down to a distance of about one-sixth of a millimetre from the end of the tracks, and their curlyness, so close to the end, makes it difficult to place much confidence in the number of *small* angle bends.

Since the observed and expected distributions are in agreement for values of ϕ greater than ϕ_m, we can proceed to calculate the velocities.

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* 126 divisions correspond to 1 cm. equivalent range at N.T.P.

Putting $N_0 = 2{\cdot}705 \times 10^{19}$; $e = 17{\cdot}1\,\epsilon$;[*] $E = 2\epsilon$; $\epsilon = 4{\cdot}77 \times 10^{-10}$; $M = 6{\cdot}56 \times 10^{-24}$; we find that

$$K_1 = 1{\cdot}03 \times 10^{8}.$$

The values 760, 288, 1·22, 1/1·67 and 176, for P, T, p, M and q respectively give

$$K_2 = 0{\cdot}295,$$

while the values of w for the four areas are found to be 2·89, 4·81, 6·94 and 17·65 respectively.

If we now express the velocities as fractions of the initial velocity of α-particles from radium *c*, which will be taken as $1{\cdot}922 \times 10^{9}$ cm. per second, we obtain the following values for v/v_0.

Table III.

$\frac{1}{2}(r_1 + r_2)$	5	11	22·5	65
v/v_0	0·047	0·078	0·112	0·287
r_0	4·7	10·4	21·5	51·5
Range (in cm.)	0·037	0·088	0·17	0·40

In the first row is given the distance of the mid-point of each area from the end of the track, and in the second row the corresponding values of v/v_0. These are plotted as dots in fig. 7. The constants of the straight line which passes roughly through the points are used to evaluate r_0 for each area, according to equation (18). The last row gives these values the equivalent range reduced to centimetres in the gas at 15° and 760 mm. As circles in fig. 6 are plotted these final values of v/v_0 and r_0 and the smooth curve through the points represents the approximate form of the function $g(r)$ Owing to the limited number of tracks considered not much weight can be given to the actual curve, but the general magnitude of the velocity of the particles at these short distances from the end of the track is unmistakable.

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[*] In the case of two mixed gases of atomic numbers z_1 and z_2 of which there are n_1 and n_2 atoms per unit volume, the total number of bends is proportional to

$$n_1 z_1^2 + n_2 z_2^2,$$

neglecting the correction due to the motion of the nucleus, which can be seen to be small. In these calculations the mixed gases were considered as simple gases of atomic number z_0 where

$$(n_1 + n_2)\, z_0^2 = n_1 z_1^2 + n_2 z_2^2.$$

For a mixture of 75 per cent. argon and 15 per cent. nitrogen $z_0 = 17{\cdot}1$.
For air $z_0 = 7{\cdot}25$.

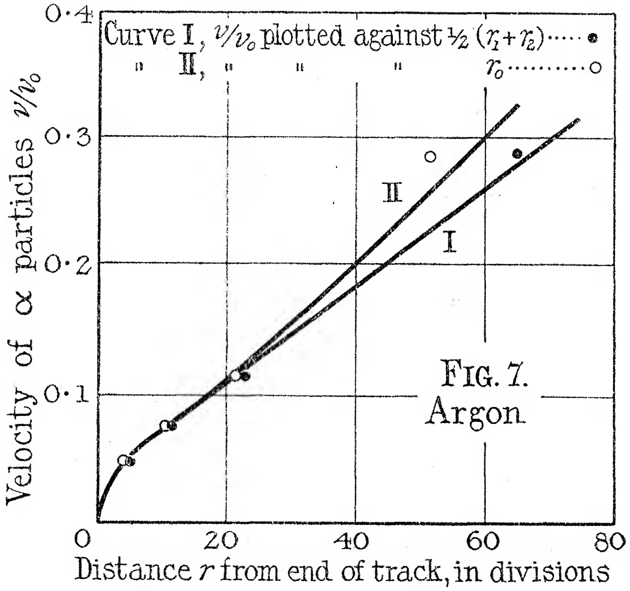

FIG. 7.
Argon.

12. *The Experiments with Air.*

The next work was carried out with tracks in air. The procedure was exactly as described for the argon tracks except that more tracks were observed, but not all of them measured up for all types of bend. The object was rather to determine the form of the function $g(r)$ than to verify the distribution with regard to angle over greater limits than were sufficient for the former purpose. Thus all bends down to the smallest observable angle were counted and measured for large values of r, while only bends over some larger angle were observed for the smaller values. The total number of tracks considered was 1524.

Putting $N = 2 \times 2{\cdot}705 \times 10^{19}$, and $e = 7{\cdot}25\,\epsilon$ we get $K_1 = 8{\cdot}08 \times 10^7$.

The measured values of P, T, p, M and q were 760, 288, 1·33, 1/1·62 and 153·3, which give $K_2 = 0{\cdot}295$.

The results of the statistical investigations are given in Table IV, and the final results of the experiment in Table V. In these Tables 126 divisions are equivalent to 1 cm. range at N.T.P.

Table IV.

ϕ_1, ϕ_2.	30, 40.	40, 60.	60, 180.
$r_1 = 3$ P	—	15	7
$r_2 = 5$ $\bar{\text{P}}$	—	16·5	5·5
$r_1 = 5$ P	12	12	5
$r_2 = 10$ $\bar{\text{P}}$	16·5	9·0	3·0

ϕ_1, ϕ_2.	15, 20.	20, 25.	25, 40.	40, 180.
$r_1 = 10$ P	15	10	9	5
$r_2 = 20$ $\bar{\text{P}}$	16·7	8·7	9·8	3·6

ϕ_1, ϕ_2.	10, 15.	15, 20.	20, 180.
$r_1 = 20$ P	11	4	7
$r_2 = 35$ $\bar{\text{P}}$	13·4	3·7	4·9
$r_1 = 35$ P	(9)	6	10
$r_2 = 60$ $\bar{\text{P}}$	25	6·9	9·1
$r_1 = 60$ P	13	11	5
$r_2 = 140$ $\bar{\text{P}}$	17·7	5·6	7·4

Table V.

r_1, r_2.	3, 5.	5, 10.	10, 20.	20, 35.	35, 60.	60, 140.
ϕ_m	40	30	20	10	15	10
P	22	29	39	22	16	29
Q	444	447	447	447	1524	1500
v/v_0	0·046	0·067	0·126	0·173	0·229	0·334
$\frac{1}{2}(r_1+r_2)$	4	7·5	15	27·5	47·5	100
r_0	3·8	7·2	14·5	27·3	46·0	91
r_0	3·8	7·0	13·9	26·2	45·6	89
Range (in cm.)	0·0302	0·0555	0·111	0·208	0·362	0·707

The value of F for this distribution is 13·7 while $n-1$ is 16, so that we conclude that the theoretical distribution is the right one.

The points marked by dots on fig. 8 show the values of v/v_0 plotted against $\frac{1}{2}(r_1+r_2)$. The curve through these points was taken as linear over the breadth of each area, and the value of A/B for each part used to calculate the values of r_0 given in the sixth row. The points given by the values of v/v_0 and r_0 were found to lie closely on a curve represented by

$$v = kr^{2/3}.$$

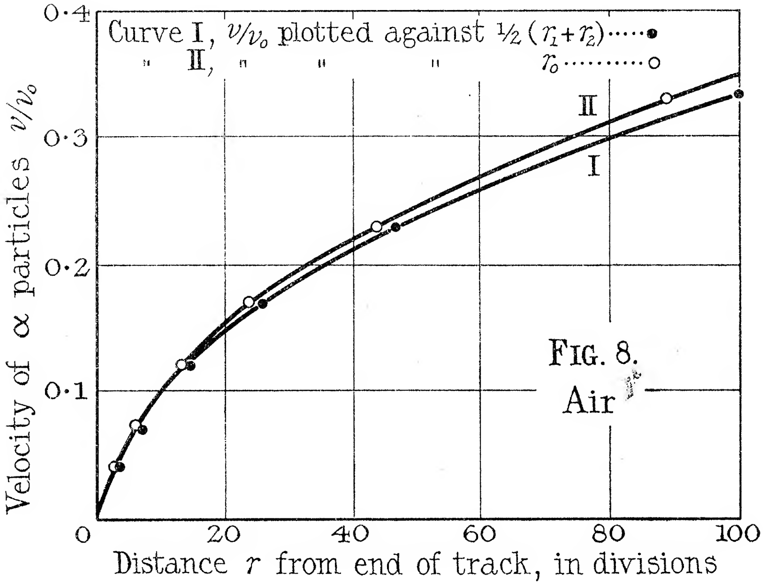

FIG. 8.
Air

Using this expression in (17), the values of r_0 in the seventh row were obtained. The close agreement with the values in the previous row showed that the process of approximation had been carried far enough. The last row gives the final values of r_0 reduced to ranges in centimetres of gas at N.T.P. The points on fig. 7 marked with a circle represent the values of v/v_0 are plotted against the final values of r_0, and the comparison of the two curves indicates the difference between the first and the last approximation to the function $g(r)$.

13. *Discussion of Results.*

The agreement that has been found in these experiments between the observed and expected distributions allows us to deduce certain conclusions about the dynamics of the collisions.

In considering the question of the degree of verification given to the assumptions on which the theory is based, it must be remembered that there will be other average distributions appreciably different from that given here, which would be equally consistent with the observations. The latitude in the choice of such theoretical distributions will of course decrease as the number of observations increase, but with such a limited number as are here available, it is only possible to specify the *general type* of average distribution. Consequently, the only assumptions that can in any way be considered as verified are those to changes of which the theoretical distribution is sensitive.

Now it has been shown by Darwin* that such a distribution is sensitive to modifications of the law of force, the substitution of an inverse cube law, for instance, markedly altering the average distribution. Thus the observed agreement can be considered as indicating the existence of a law of force not much different from that of the inverse square. But it is less easy to decide to what degree the energy and momentum assumptions are verified. By far the most delicate test of these last is obtained by the study of individual branched tracks, into the consideration of which the law of force does not enter.

As to the question of the region in which the existence of inverse-square law is supported, it is necessary to calculate the maximum and minimum apsidal distances* of the orbits corresponding to the deflections considered. In the case of both argon and air, the most intimate orbits are those in which an α-particle of velocity about $0{\cdot}45\,v_0$ is deflected through an angle of about 30°, while the least intimate are those in which particles of velocity $0{\cdot}04\,v_0$ are deflected through 60° and 40° respectively. The apsidal distances in these cases are about $7{\cdot}4 \times 10^{-12}$ and $5{\cdot}4 \times 10^{-10}$ cm. for argon and $3{\cdot}2 \times 10^{-12}$ and $3{\cdot}7 \times 10^{-10}$ cm. for air. And since an orbit is appreciably determined by the forces at distances considerably greater than the apsidal distance, it can be said that these experiments support the hypothesis that the inverse-square law holds between the nucleus and an α-particle when their distance apart lies between about 7×10^{-12} and 10^{-9} cm. in the case of argon and 3×10^{-12} and 5×10^{-10} cm. in the case of air. It is interesting to note that the upper limit for argon is considerably greater than the distance of the K ring electrons from the nucleus. From the results, however, it appears possible that, especially in the case of argon, there is a departure from the inverse-square law at distances greater than those given.

It will be noticed that no confirmation of the magnitude of the nuclear change can be obtained from these results unless the velocities here calculated are independently confirmed. But the discovery that the velocities in air and argon are found to be almost identical close to the end of the range, in spite of the fact that bends in general are about three times more frequent in argon than in air, does indirectly confirm the assumption. For the range in argon of an α-particle from polonium has been shown by Adams† to be only 12 per cent. greater than in air, so that it would be expected that the relation between the velocity and the range would also be similar.

The analysis reveals the existence of α-particles of very low velocity compared with any that had been observed before. It appears that α-particles at a distance of about a quarter of a millimetre from the end of the track have a velocity about $0 \cdot 04 v_0$. Now it can easily be seen that this is just about the minimum velocity which must be possessed by an α-particle in order that it should impart to a free electron energy corresponding to a fall through 13·5 volts. In other words, the minimum velocity measured is roughly that at which the α-particle may be expected to lose its ionising power in air.

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* Darwin, *loc. cit.*

† Adams, 'Phys. Rev.,' vol. 24, p. 113 (1907).

There is considerable difficulty in comparing these results with the data obtained by other methods, for the reason that these results refer to the average velocity of a number of α-particles at a given distance from the end of the track, while all other results refer to the average properties of a beam of α-rays at a given distance from the source. However, Marsden and Taylor* have shown that the function $g(r)$ is of the form $r^{1/3}$ when the absorbing material is air, and r is greater than about 2 cm., while both their work with heavy metals as absorbing material and the theoretical investigation of Bohr suggest that for lower velocities the speed of the particles will be proportional to some higher power of r.

The present results are fully in accord with this view since, as has already been mentioned, the relation

$$v = kr^{2/3}$$

appears roughly to represent the observations, which are all for values of r less than 1 cm. But it does not seem possible without further data to discuss usefully the transition between these two approximate impressions for $g(r)$. For it must be remembered that the statistical investigations are based on the assumption that an α-particle of given velocity will travel a given distance. If, as has recently been suggested by Henderson,† a large degree of straggling occurs in the last few millimetres of the range, this assumption will be only very approximately true. But though it is unlikely that the neglect of this phenomenon in the analysis can seriously affect the order of magnitude of the calculated velocities, it must certainly instil caution against attaching undue weight to the exact form discovered for the relation between the velocity and the remaining range.

In conclusion, it may be said that the recent measurements by Kapitza‡ of the heating effect of a beam of α-rays give, for the average velocity at a given distance from the *end of the beam*, values nearly the same as those given by the statistical method for the average velocity at a given distance from the

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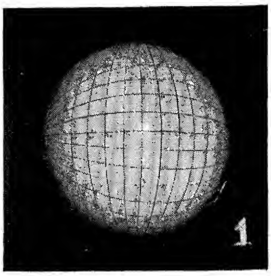
* Marsden and Taylor, 'Roy. Soc. Proc.,' A, vol. 88, p. 443 (1913).

† Henderson, 'Phil. Mag.,' vol. 42, p. 538 (1921).

‡ Kapitza, 'Roy. Soc. Proc.,' vol. 102, p. 48 (1922).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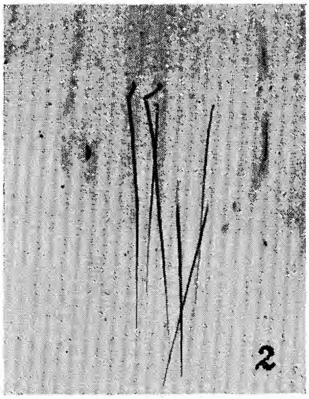
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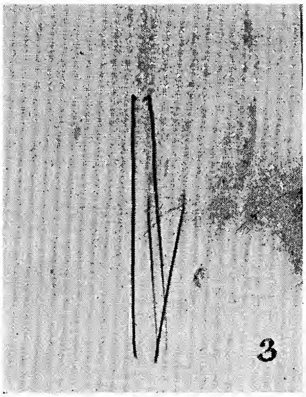
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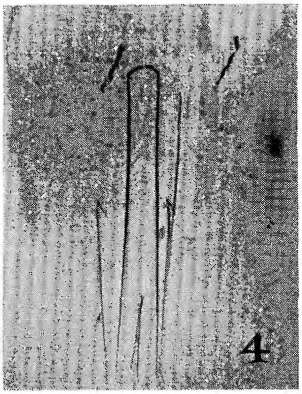
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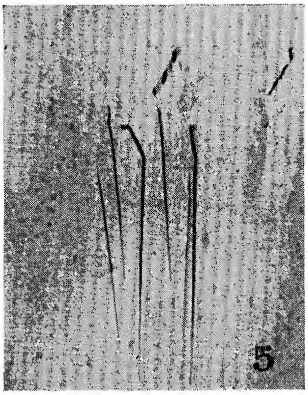
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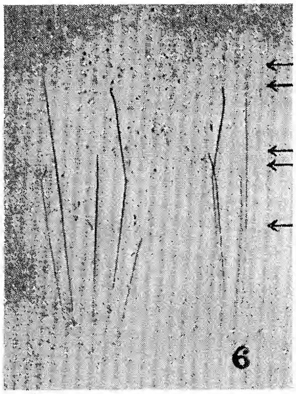
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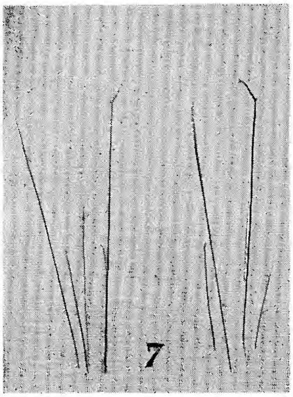
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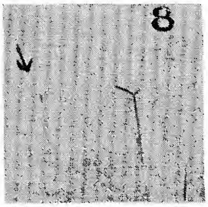
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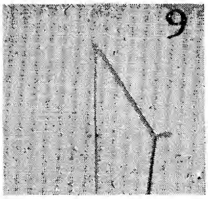
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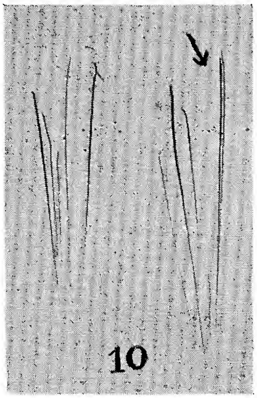
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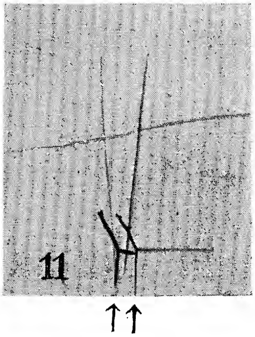
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end of each track. The actual values are shown below, and refer to air at 15° and 760 mm. :—

Distance from source (in cm.)	6·26	6·50	6·72	7·07
Distance from end of beam (in cm.)	0·81	0·57	0·35	0
Heating effect (Kapitza)	0·400	0·316	0·244	0·084
Frequency of bends	0·365	0·300	0·224	0

If no straggling occurred this agreement would be expected. In view of the large degree of straggling which does occur, it is a little difficult to account either for this agreement or for the disagreement which would be shown if the straggling were to be taken into account.

14. *Summary.*

1. Mr. C. T. R. Wilson's expansion method was used to study in detail the passage of α-rays through air and argon, attention being especially directed to the latter part of the range.

2. The frequency of large angle deflections was found from the study of the photographs of a large number of tracks, and compared with that to be expected on the nuclear theory of the atom.

3. The agreement found indicates that the inverse square law holds between the nuclei and the α-particle, when their distance apart lies between about 7×10^{-12} and 10^{-9} cm. for argon, and 3×10^{-12} and 5×10^{-10} cm. for air.

4. The velocity of the α-particle along the latter part of its range was determined from the frequency of bends and found to be as low as $v_0/20$ very near the end, where v_0 is the velocity of the α-particles from radium C.

5. No anomalous effect was discovered either as regards frequency or type of collision.

My thanks are due to Sir Ernest Rutherford for his continual interest and advice throughout this work.

DESCRIPTION OF PLATE 2.

All these reproductions are 3·06 times larger than the photographs.

Nos. 2 to 5 are of tracks in argon, for which $p = 1·22$, $M = 1/1·67$. Hence these tracks are 2·24 times larger than they would be in the gas at 15°, 760 mms.

Nos. 6 to 11 are of tracks in air, for which $p = 1·33$, $M = 1/1·62$. Hence these tracks are 2·51 times as large as they would be at 15°, 760 mms.

In all the photographs 1 cm. corresponds to about 60 divisions of the microscopic scale.

No. 2.—The long track shows a bend (75°, 23) followed by a short hook near the end.

No. 3.—A single (109°, 8) bend.

No. 4.—A (63°, 17) followed by a (48°, 6) on the long track. The two images have their ends nearly super-imposed. Also a (140°, 16) lower down the photo. Note the short "spur" to this latter and compare it with the long spur due to the lighter "air" atom in No. 10.

No. 5.—A (31°, 26) followed by an (80°, 7).

No. 6.—One track showing an early bend (20°, 65) with a small spur.

No. 7.—A (42°, 19) bend also showing spur.

No. 8.—A (52°, 16) with a long spur. The left-hand image marked by an arrow shows that the three parts of the track are co-planar.

No. 9.—An early bend (39°, 18) with long spur. The unusual breadth of the r.-h. image is probably an optical effect.

No. 10.—A marked bend (132°, 18) indicating a nearly head-on collision. The r.-h. image is marked by an arrow.

No. 11.—A remarkable track showing an (89·5°, 59) bend. The direction of the initial part of the track upwards. The l.-h. image shows the remaining part of the track very much foreshortened. The long spur shows two marked bends, evidently due to collision of the recoil atom itself with two atoms of the gas. This track is the most suitable for accurate measurement, and for it the mass ratio m/M is found to be 3·3. Almost certainly then the atom collided with is a nitrogen atom.

On the Structure and Chemical Activity of Copper Films, and the Colour Changes accompanying their Oxidation.

By C. N. Hinshelwood, B.A., Fellow of Trinity College, Oxford.

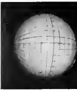
(Communicated by Prof. J. W. Nicholson, F.R.S. Received August 30, 1922.)

When the surface of a metal is exposed to the action of a gas with which it reacts chemically, brilliant colour phenomena are frequently produced. In some cases the colours are recognised as diffraction colours, produced by the scattering of light in the surface film, and not as simple interference effects. When this is found the film must have a more or less complex structure, fine grained compared with the wave-length of light, but of a coarse-grained granular nature compared with molecular magnitudes. The work of Beilby has drawn attention from other points of view to the complex structure which the surface layers of metals, or thin films of metals, may assume.

It seems to be of considerable importance to correlate the chemical activity of such surface films with their structure, in view of the bearing this correlation may have on the problems of heterogeneous catalysis.

A first attempt in this direction is made in this paper, the reaction investigated being the oxidation and reduction of a copper—copper oxide film on the surface of metallic copper.

The general nature of the phenomena observed is as follows: When bright,



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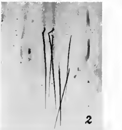

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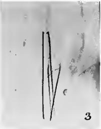

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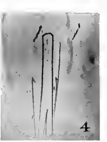

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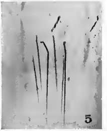

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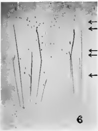

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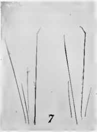

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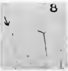

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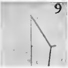

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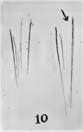

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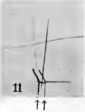

11

DESCRIPTION OF PLATE 2.

All these reproductions are 3·06 times larger than the photographs.

Nos. 2 to 5 are of tracks in argon, for which $p = 1{\cdot}22$, $M = 1/1{\cdot}67$. Hence these tracks are 2·24 times larger than they would be in the gas at 15°, 760 mms.

Nos. 6 to 11 are of tracks in air, for which $p = 1{\cdot}33$, $M = 1/1{\cdot}62$. Hence these tracks are 2·51 times as large as they would be at 15°, 760 mms.

In all the photographs 1 cm. corresponds to about 60 divisions of the microscopic scale.

No. 2.—The long track shows a bend (75°, 23) followed by a short hook near the end.

No. 3.—A single (109°, 8) bend.

No. 4.—A (63°, 17) followed by a (48°, 6) on the long track. The two images have their ends nearly super-imposed. Also a (140°, 16) lower down the photo. Note the short "spur" to this latter and compare it with the long spur due to the lighter "air" atom in No. 10.

No. 5.—A (31°, 26) followed by an (80°, 7).

No. 6.—One track showing an early bend (20°, 65) with a small spur.

No. 7.—A (42°, 19) bend also showing spur.

No. 8.—A (52°, 16) with a long spur. The left-hand image marked by an arrow shows that the three parts of the track are co-planar.

No. 9.—An early bend (39°, 18) with long spur. The unusual breadth of the r.-h. image is probably an optical effect.

No. 10.—A marked bend (132°, 18) indicating a nearly head-on collision. The r.-h. image is marked by an arrow.

No. 11.—A remarkable track showing an (895°, 59) bend. The direction of the initial part of the track upwards. The l.-h. image shows the remaining part of the track very much foreshortened. The long spur shows two marked bends, evidently due to collision of the recoil atom itself with two atoms of the gas. This track is the most suitable for accurate measurement, and for it the mass ratio m/M is found to be 3·3. Almost certainly then the atom collided with is a nitrogen atom.